

Large Effects of CP- and T-Violation in K^0 Decay

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The ϵ -impurity of the K_L wave-function gives rise to a huge CP -violating effect in the decay $K_L \rightarrow \pi^+\pi^-\gamma$ which is hidden in the polarization state (Stokes vector) of the photon. One component of the Stokes vector is CP -odd and T -even, and may be identified with circular polarization. Another component is CP -odd and T -odd (“oblique polarization”) and reveals itself as a large asymmetry (14%) in the decay $K_L \rightarrow \pi^+\pi^-e^+e^-$. Striking time-dependent effects are predicted in the angular distribution of the $\pi^+\pi^-e^+e^-$ system emanating from an initial K^0 or \bar{K}^0 state.

It is not customary to use the word “large” in association with CP -violating effects in K^0 decays. Nevertheless, a large effect has been observed in the decay $K_L \rightarrow \pi^+\pi^-e^+e^-$ [1,2], in agreement with theoretical predictions [3,4]. In this talk, I explain the origin of this effect, and some of its ramifications.

1. CP - and T -Violation in $K_L \rightarrow \pi^+\pi^-\gamma$

The decay $K_S \rightarrow \pi^+\pi^-\gamma$ is known to be well-described by pure bremsstrahlung. By contrast, the branching ratio and the photon energy spectrum of $K_L \rightarrow \pi^+\pi^-\gamma$ require a mixture of bremsstrahlung and direct $M1$ emission [5,6]. A simple ansatz for the matrix elements is [7]

$$\begin{aligned} \mathcal{M}(K_{S,L} \rightarrow \pi^+\pi^-\gamma) = & \frac{e|f_S|}{M_K^4} \{E_{S,L}(\omega, \cos\theta) [\epsilon \cdot p_+ k \cdot p_- - \epsilon \cdot p_- k \cdot p_+] \\ & + M_{S,L}(\omega, \cos\theta) \epsilon_{\mu\nu\rho\sigma} \epsilon^\mu k^\nu p_+^\rho p_-^\sigma\}, \end{aligned} \quad (1)$$

where

$$\begin{aligned} E_S &= \left(\frac{2M_K}{\omega}\right)^2 \frac{e^{i\delta_0(s=M_K^2)}}{1 - \beta^2 \cos^2\theta}, \quad M_S = 0, \\ E_L &= \left(\frac{2M_K}{\omega}\right)^2 \frac{\eta_{+-} e^{i\delta_0(s=M_K^2)}}{1 - \beta^2 \cos^2\theta}, \\ M_L &= i(0.76)e^{i\delta_1(s)}. \end{aligned} \quad (2)$$

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Here ω is the photon energy in the K_L rest frame, θ the angle between π^+ and γ in the $\pi^+\pi^-$ c.m. frame, and $\beta = \sqrt{1 - \frac{4m_\pi^2}{s}}$, s being the $\pi^+\pi^-$ invariant mass. The coefficient 0.76 in M_L is determined from the empirical strength of direct emission in $K_L \rightarrow \pi^+\pi^-\gamma$ [3]. The phase factor $e^{i\delta_0(M_K^2)}$ in the bremsstrahlung amplitudes $E_{L,S}$ is dictated by the Low theorem, while the factor $i e^{i\delta_1(s)}$ in M_L is determined by CPT invariance and the Watson theorem. The important feature of the K_L amplitude is that the bremsstrahlung component E_L , proportional to η_{+-} , is enhanced by the factor $(2M_K/\omega)^2$, making it comparable to the direct emission amplitude M_L . The interference of the electric multipole ($CP = +1$) with the magnetic multipole ($CP = -1$) opens the way to large CP -violating observables. Such interference effects vanish, however, if one sums over the photon polarization. Thus CP violation is encrypted in the polarization state if the photon.

The polarization state of the photon can be described by the Stokes vector $\vec{S} = (S_1, S_2, S_3)$ whose components are (dropping the subscript L in E_L, M_L)

$$\begin{aligned} S_1 &= \frac{2\text{Re}E^*M}{|E|^2 + |M|^2}, \\ S_2 &= \frac{2\text{Im}E^*M}{|E|^2 + |M|^2}, \end{aligned}$$

$$S_3 = \frac{|E|^2 - |M|^2}{|E|^2 + |M|^2}. \quad (3)$$

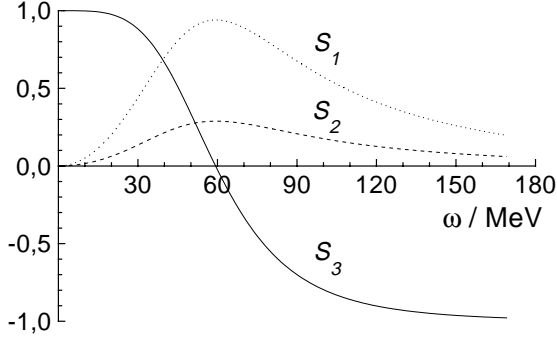


Figure 1. Stokes parameters of the photon in $K_L \rightarrow \pi^+\pi^-\gamma$

These are plotted in Fig. 1 as a function of the photon energy. The components S_1 and S_2 are CP -violating observables, and are remarkably large, considering that they originate in the small parameter $\epsilon \approx \eta_{+-}$. To see the physical meaning of these parameters, we choose a frame in which $\vec{k} = (0, 0, \omega)$ and $\vec{n}_\pi = (\vec{p}_+ \times \vec{p}_-)/|\vec{p}_+ \times \vec{p}_-| = (1, 0, 0)$. The parameter S_2 is then recognized as the *circular polarization*, i.e.

$$S_2 = [d\Gamma(L) - d\Gamma(R)]/[d\Gamma(L) + d\Gamma(R)] \quad (4)$$

where L and R refer to the polarization vectors $\vec{\epsilon}_{R,L} = (1, \pm i, 0)/\sqrt{2}$. This is a CP -odd but T -even observable, and vanishes in the hermitian limit $\delta_0 = \delta_1 = 0$, $\arg\eta_{+-} = \pi/2$ [7]. To understand the significance of S_1 , we consider the linear polarization vector $\vec{\epsilon} = (\cos\phi, \sin\phi, 0)$, where ϕ is the polarization direction relative to \vec{n}_π , the normal to the decay plane. One then discovers that S_1 is the “*oblique polarization*”, defined as the difference between the decay rates for $\phi = 45^\circ$ and $\phi = 135^\circ$:

$$S_1 = \frac{d\Gamma(45^\circ) - d\Gamma(135^\circ)}{d\Gamma(45^\circ) + d\Gamma(135^\circ)}. \quad (5)$$

More generally, the decay rate as a function of ϕ is

$$\frac{d\Gamma}{d\phi} \sim 1 - [S_3 \cos 2\phi + S_1 \sin 2\phi]. \quad (6)$$

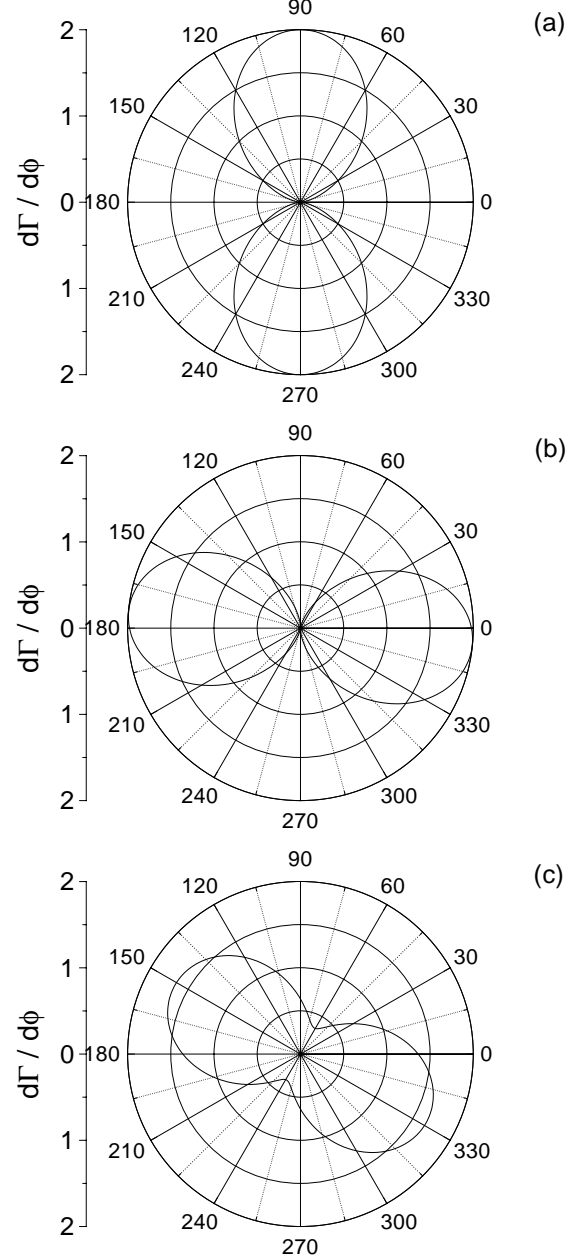


Figure 2. Distribution of $K_L \rightarrow \pi^+\pi^-\gamma$ in angle ϕ between polarization vector $\vec{\epsilon}$ and normal to decay plane. (a) $\omega \rightarrow 0$, (b) $\omega \rightarrow 170 \text{ MeV}$, (c) average over $20 \text{ MeV} < \omega < 170 \text{ MeV}$.

Fig. 2 illustrates how this pattern changes from

an electric distribution $d\Gamma/d\phi \sim \sin^2 \phi$ at low ω to a magnetic distribution $d\Gamma/d\phi \sim \cos^2 \phi$ as the photon energy increases. The presence of the parameter S_1 produces a tilted pattern in which there is an asymmetry between the quadrants I+III compared to II+IV. It is this tilt that signals a violation of CP . By examining the behaviour of \vec{k} , $\vec{\epsilon}$ and \vec{n}_π und CP and T , we find that $\sin 2\phi$ transforms as $CP = -, T = -$. Thus the oblique polarization S_1 is a CP -odd, T -odd observable [7]. Unlike the parameter S_2 , the parameter S_1 survives in the hermitian limit. In this sense, the T -odd property of S_1 is not an artifact of dynamical phases, but rather an example of T -violation accompanying CP -violation in a CPT invariant theory [7]. The study of the Dalitz pair reaction $K_L \rightarrow \pi^+\pi^-\gamma$ may be viewed as an attempt to detect the oblique polarization of the photon in $K_L \rightarrow \pi^+\pi^-\gamma$, using the plane of the e^+e^- pair as an analyser.

2. CP - and T -Violation in $K_L \rightarrow \pi^+\pi^-\gamma$

The matrix element of the reaction $K_L \rightarrow \pi^+\pi^-\gamma$ can be written as [3,4]

$$\mathcal{M} = \mathcal{M}_{br} + \mathcal{M}_{mag} + \mathcal{M}_{CR} + \mathcal{M}_{SD} \quad (7)$$

where the \mathcal{M}_{br} and \mathcal{M}_{mag} are associated with the bremsstrahlung and $M1$ components of the radiative amplitude. The term \mathcal{M}_{CR} denotes a “charge radius” contribution, corresponding to $\pi^+\pi^-$ emission in an s -wave, not possible for the real radiative process $K_L \rightarrow \pi^+\pi^-\gamma$. The term \mathcal{M}_{SD} represents the contribution of the short-distance interaction $s\bar{d} \rightarrow e^+e^-$. Estimates in [3,4] showed that the amplitude is dominated by the first two terms in Eq.(7), as is now borne out by the data [1,2]. The differential decay rate may be calculated in the form

$$d\Gamma = I(s_\pi, s_l, \cos \theta_l, \cos \theta_\pi, \phi) ds_\pi ds_l d\cos \theta_l d\cos \theta_\pi d\phi \quad (8)$$

where $s_\pi(s_l)$ is the invariant mass of the pion (lepton) pair and $\theta_\pi(\theta_l)$ is the angle of the $\pi^+(l^+)$ in the $\pi^+\pi^-(l^+l^-)$ rest frame, relative to the dilepton (dipion) direction. For the purpose of detecting the oblique polarization in $K_L \rightarrow \pi^+\pi^-\gamma$, the

relevant variable is ϕ , the angle between the normals to the $\pi^+\pi^-$ and l^+l^- planes. Defining the unit vectors

$$\begin{aligned} \vec{n}_\pi &= \frac{\vec{p}_+ \times \vec{p}_-}{|\vec{p}_+ \times \vec{p}_-|}, \\ \vec{n}_l &= \frac{\vec{k}_+ \times \vec{k}_-}{|\vec{k}_+ \times \vec{k}_-|}, \\ \vec{z} &= \frac{\vec{p}_+ + \vec{p}_-}{|\vec{p}_+ + \vec{p}_-|}, \end{aligned} \quad (9)$$

we have

$$\begin{aligned} \sin \phi &= \vec{n}_\pi \times \vec{n}_l \cdot \vec{z} \quad (CP = -, T = -) \\ \cos \phi &= \vec{n}_l \cdot \vec{n}_\pi \quad (CP = +, T = +). \end{aligned} \quad (10)$$

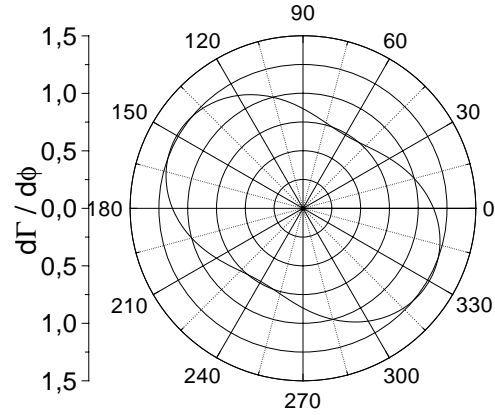


Figure 3. Distribution of $K_L \rightarrow \pi^+\pi^-\gamma$ in angle ϕ between $\pi^+\pi^-$ and e^+e^- planes.

Integrating over all variables other than ϕ , one obtains [7]

$$\frac{d\Gamma}{d\phi} \sim 1 - (\Sigma_3 \cos 2\phi + \Sigma_1 \sin 2\phi) \quad (11)$$

with $\Sigma_3 = -0.133$ and $\Sigma_1 = 0.23$. This distribution is plotted in Fig. 3, and shows clearly the CP - and T -violating “tilt” similar to the oblique polarisation in Fig. 2. The KTeV and NA48 experiments confirm the predicted ϕ -distribution [1, 2], and also the integrated asymmetry

$$\mathcal{A}_\phi = \frac{\left(\int_0^{\pi/2} - \int_{\pi/2}^\pi + \int_\pi^{3\pi/2} - \int_{3\pi/2}^{2\pi} \right) \frac{d\Gamma}{d\phi}}{\left(\int_0^{\pi/2} + \int_{\pi/2}^\pi + \int_\pi^{3\pi/2} + \int_{3\pi/2}^{2\pi} \right) \frac{d\Gamma}{d\phi}}$$

$$= -15\% \sin(\delta_0 - \delta_1 + \varphi_{+-}) = -14\%. \quad (12)$$

3. Time-Evolution of the Decay Spectrum in $K^0(\bar{K}^0) \rightarrow \pi^+\pi^-\gamma$ [8]

Consider the decay $K^0(\bar{K}^0) \rightarrow \pi^+\pi^-\gamma$ of a state that is prepared as an eigenstate of strangeness $+1(-1)$. The decay amplitude at a subsequent time t can be expressed in terms of the amplitudes $E_{L,S}$, $M_{L,S}$ as follows:

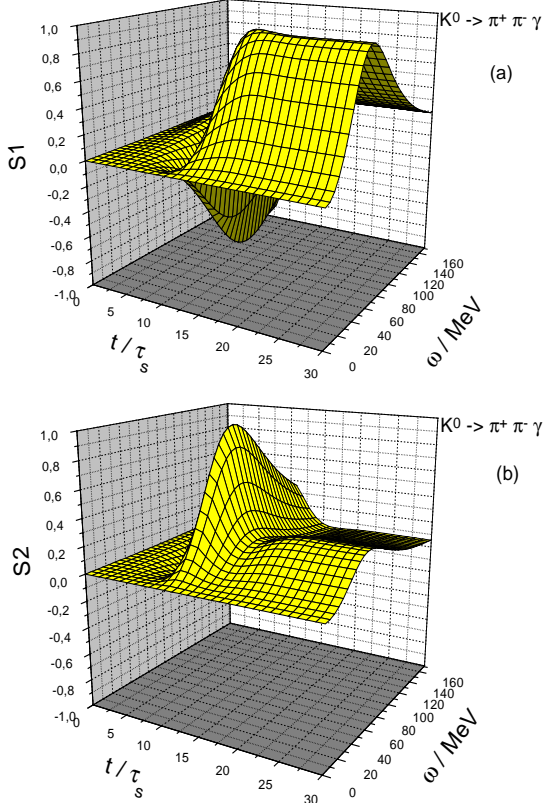


Figure 4. Components S_1 and S_2 of the Stokes vector of the photon as a function of photon energy and time for the decay $K^0 \rightarrow \pi^+\pi^-\gamma$.

$$\mathcal{M}(K^0(t) \rightarrow \pi^+\pi^-\gamma) \sim \{E(t, \omega, \cos \theta) [\epsilon \cdot p_+ k \cdot p_- - \epsilon \cdot p_- k \cdot p_+]$$

$$+ M(t, \omega, \cos \theta) \epsilon_{\mu\nu\rho\sigma} \epsilon^\mu k^\nu p_+^\rho p_-^\sigma \},$$

$$\begin{aligned} \mathcal{M}(\bar{K}^0(t) \rightarrow \pi^+\pi^-\gamma) \sim \\ \{ \bar{E}(t, \omega, \cos \theta) [\epsilon \cdot p_+ k \cdot p_- - \epsilon \cdot p_- k \cdot p_+] \\ + \bar{M}(t, \omega, \cos \theta) \epsilon_{\mu\nu\rho\sigma} \epsilon^\mu k^\nu p_+^\rho p_-^\sigma \} \end{aligned} \quad (13)$$

where

$$\begin{aligned} E &= e^{-i\lambda_S t} E_S(\omega, \cos \theta) + e^{-i\lambda_L t} E_L(\omega, \cos \theta), \\ M &= e^{-i\lambda_L t} M_L(\omega, \cos \theta), \\ \bar{E} &= e^{-i\lambda_S t} E_S(\omega, \cos \theta) - e^{-i\lambda_L t} E_L(\omega, \cos \theta), \\ \bar{M} &= -e^{-i\lambda_L t} M_L(\omega, \cos \theta) \end{aligned} \quad (14)$$

with $\lambda_{L,S} = m_{L,S} - \frac{i}{2}\Gamma_{L,S}$.

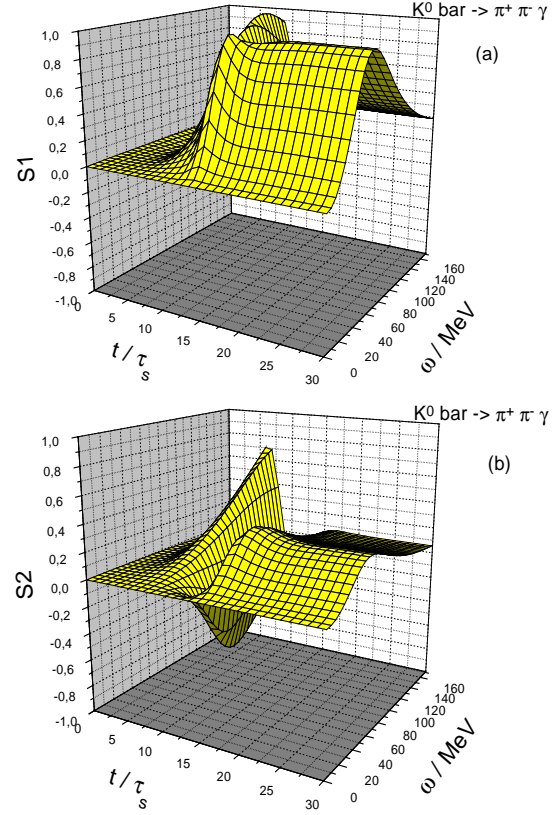


Figure 5. Components S_1 and S_2 of the Stokes vector of the photon as a function of photon energy and time for the decay $\bar{K}^0 \rightarrow \pi^+\pi^-\gamma$.

The amplitudes in Eq.(13) allow us to determine the Stokes vector of the photon at any time t . As an example, Figs. 4, 5 show the components $S_1(t)$ and $S_2(t)$ as function of energy for an initial K^0 or \bar{K}^0 . It is interesting to ask how this time dependence would be reflected in the decay spectrum of $K^0(\bar{K}^0) \rightarrow \pi^+\pi^-e^+e^-$.

This question has been analysed in [8]. In particular, we have calculated the time-dependent correlation of the $\pi^+\pi^-$ and e^+e^- planes,

$$\frac{d\Gamma}{d\phi} \sim 1 - (\Sigma_3(t) \cos 2\phi + \Sigma_1(t) \sin 2\phi), \quad (15)$$

and the associated asymmetry $\mathcal{A}_\phi(t)$ defined as in Eq.(12).

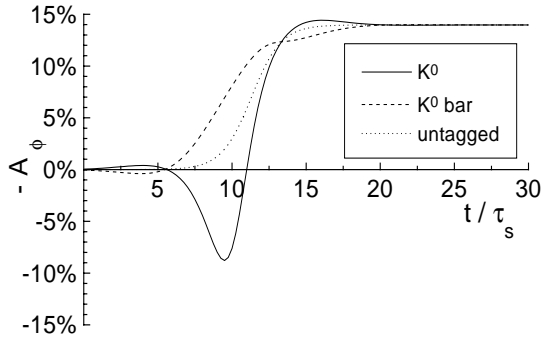


Figure 6. Time-dependent asymmetry \mathcal{A}_ϕ for the decays K^0 and $\bar{K}^0 \rightarrow \pi^+\pi^-e^+e^-$ as well as for an incoherent equal mixture.

The result is displayed in Fig. 6, and shows a remarkable time variation, that differs between K^0 and \bar{K}^0 . As expected (and as measured by NA48 [2]), the asymmetry vanishes at short times, where the K meson decays essentially as K_S (and the amplitude of $K_S \rightarrow \pi^+\pi^-\gamma$ is purely electric). At large times the asymmetry approaches the asymptotic value $\mathcal{A}_\phi = -14\%$ expected for $K_L \rightarrow \pi^+\pi^-e^+e^-$. Also shown in Fig. 6 is the result to be expected if the source of K -mesons is an untagged equal mixture of K^0 and \bar{K}^0 (such as derived from $\phi \rightarrow K^0\bar{K}^0$ at DAΦNE). The non-zero value of $\mathcal{A}_\phi(t)$ for such an untagged beam represents a CP - and T -violating effect at any decay time.

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REFERENCES

1. KTeV Collaboration, A. Alavi-Harati et al., Phys. Rev. Lett. 84 (2000) 408; B. Cox, these Proceedings.
2. NA48 Collaboration, E. Mazzucato, these Proceedings.
3. L.M. Sehgal, M. Wanninger, Phys. Rev. D 46 (1992) 1035; 46 (1992) 5209(E).
4. P. Heiliger, L.M. Sehgal, Phys. Rev. D 48 (1993); 60 (1999) 079902(E).
5. E.J. Ramberg et al., Fermilab Report No. FERMILAB-CONF-91-258, 1991; E731 Collaboration, Phys. Rev. Lett. 70 (1993) 2525.
6. KTeV Collaboration, A. Alavi-Harati et al., hep-ex/0008045.
7. L.M. Sehgal, J. van Leusen, Phys. Rev. Lett. 83 (1999) 4933.
8. L.M. Sehgal, J. van Leusen, Phys. Lett. B 489 (2000) 300.